

Lecture 15

Tuesday, February 16, 2021 2:24 PM

- * Prayer
 - * Spiritual thought
 - * Answering questions
-

Chain rule

$$z = f(x, y) = f(x(t), y(t))$$

Ex

$$z = f(x, y) = xy$$



$$x = t^2$$

$$y = t$$

what is $\frac{dz}{dt}$?

$$\Delta z = x \Delta y + y \Delta x$$

$$z' = xy' + yx'$$

Recall: $[f(g(x))]' = \underbrace{f'(g(x))}_{\text{der. of the "outer" function}} \underbrace{g'(x)}_{\text{der. of the "inner" functions}}$

$$\Delta f(y) \approx f'(y) \Delta y \approx f'(y) y' \Delta x$$

Now $f = f(x, y)$

$$\Delta f \approx f_x \Delta x + f_y \Delta y \approx f_x x' \Delta t + f_y y' \Delta t$$

$$\implies \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$f(x, y) = f(x(u, v, w), y(u, v, w))$$

$$df = f_x dx + f_y dy = f_x (x_u du + x_v dv + x_w dw) + f_y (y_u du + y_v dv + y_w dw)$$

$$= (f_x x_u + f_y y_u) du + (f_x x_v + f_y y_v) dv + (f_x x_w + f_y y_w) dw$$

Thus,

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u}$$

$$\frac{\partial f}{\partial v} = \dots$$

$$\frac{\partial f}{\partial w} = \dots$$

Ex:

$$z = (x+y)^4$$

$$x = s^2 t$$

$$y = t + s$$

Find $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial s}$.

Ex:

$$z = \ln \sqrt{x^2 + y^2 + z^2}$$

$$x = t^2$$

$$y = ts$$

$$z = s^2$$

Find $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial s}$.

* Implicit differentiation

$$x^2 + yz + z^2x = 1 \quad (*)$$

$$x=1, y=-2$$

$$z = z(x, y)$$

$$\leadsto z^2 - 2z = 0$$

$$\leadsto z=0 \text{ or } z=2$$

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ at $(1, -2)$ (with $z=0$)

Differentiating (*) w.r.t x :

$$2x + yz_x + 2z z_x x + z^2 = 0 \quad \leadsto z_x = 1$$

↓	↓	↓	↓	↓
1	-2	0	1	0

Ex

$$y \sin x = x^2 - y^2 + 1$$

$$\left. \frac{dy}{dx} \right|_{x=0, y=1} = ?$$